Roll No. .... Total Pages : 3

MDE/M-16 **4255**

QUANTUM MECHANICS

Paper–I

Time : Three Hours] [Maximum Marks : 55

Note : Attempt *five* questions in all, selecting *one* question from each unit. Question No. 1 is compulsory.

Compulsory Question

1. (a) Show that operator e^{iH} is unitary if H operator is Hermitian. 2

(b) Differentiate Schrödinger, Heisenberg and Interaction pictures. 2

(c) Why can't an angular momentum vector remain fixed in one direction in space? 2

(d) Under what conditions, the perturbation theories are helpful to correct the energy eigenvalues of a particular quantum mechanical system ? 2

(e) Define Green's function. Give its importance in scattering theory. 3

UNIT–I

2. (a) Generate n^{th} excited state from ground state of Linear Harmonic Oscillator with the help of suitable operators. Find out the matrices for annihilation, creation and number operators of Linear Harmonic Oscillator. 7

(b) What do you understand by Hilbert space? Justify its role for explanation of quantum mechanical objects using suitable diagram. 4

3. (a) Using spherical polar coordinates, separate out the angular and radial part of Schrödinger wave equation for the hydrogen atom. 7

(b) What is unitary transformation? Transform the Hamiltonian operator in coordinate representation into a diagonal matrix using unitary matrix U. 4

UNIT–II

4. (a) What are the properties of Clebsch Gordan (G.G.) coefficients? Find out the C.G. coefficients for $j_1 = 1$, $j_2 = ½$. 5½

(b) Show that the angular momentum operators are the infinitesimal rotation operators. 5½

5. (a) With the help of commutation rules of angular momenta, find out the eigenvalues of the angular momentum operator. 5½

(b) Derive spin wave functions for a system of two spin ½ particles. 5½

UNIT–III

6. (a) What do you mean by Born Approximation? Approximate the Scattering Amplitude f up to the first Born Approximation. 7

(b) Explain Differential and Total Scattering Cross Section with the help of suitable diagram. 4

7. (a) Using the relation,

$$f(\theta) = \frac{1}{k}\sum_{0}^{\infty}(2l+1)e^{i\delta_l}\sin\delta_1\, P_1(\cos\theta),$$

obtain the expression for total scattering cross-section for elastic scattering. 7

(b) State and prove optical theorem. 4

UNIT–IV

8. (a) Using time, independent perturbation theory to find out the first order corrections in energy and wavefunction of a non degenerate level. 7

(b) Starting from symmetry property of the wave functions, define Fermions and Bosons. 4

9. (a) Work out to find the splitting of energy levels of hydrogen atom under the influence of uniform time independent electric field. 7

(b) Differentiate : (i) Degenerate and Non-degenerate levels (ii) first order and second order perturbation. 4

Roll No. .... Total Pages : 2

MDE/M-16 **4256**

ELECTROMAGNETIC THEORY

Paper–II

Time : Three Hours] [Maximum Marks : 55

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each unit.

Compulsory Question

1. (a) Explain the cause of atmospheric whistlers. 3

(b) How temperature can affect the performance of an antenna ? 2

(c) Define Efficiency of an antenna. 2

(d) Explain Magnetic susceptibility. 2

(e) Develop an equivalent circuit of TE waves in a wave guide. 2

UNIT–I

2. (a) Obtain the energy density & Poynting vector of a electromagnetic field. 5

(b) Determine the power loss in a plane conductor. 5

3. (a) Explain wave propagation in good dielectrics and good conductors. 7

(b) Explain the term Surface impedance. Hence determine this quantity for good conductors. 4

UNIT–II

4. (a) What do you understand by Q of a cavity ? 4

(b) Can a TEM mode exist in a hollow wave guide ? Justify your arguments. 7

5. (a) Discuss the motion of a charged particle in the presence of crossed electric and magnetic field. 7

(b) What do you mean by Attenuation in a wave guide ? 4

UNIT–III

6. (a) What are the different approaches of determining suitable potential for the electromagnetic field ? Explain *one* such technique. 7

(b) What do you understand by Centre fed linear antenna ? 4

7. (a) Determine the expression of the electric field strength for the radiation field of a half-wave dipole. 7

(b) Give a detailed account of Binomial array. 4

UNIT–IV

8. (a) Briefly describe about Ionosphere. 6

(b) Define Faraday rotation, and hence find the tilt angle of the plane of polarization. 5

9. Explain in detail the reflection and refraction of radio waves by the ionosphere. How one can determine critical frequencies and virtual heights experimentally ? 11

Roll No. .... Total Pages : 3

MDE/M-16 **4258**

CONDENSED MATTER PHYSICS AND NANO TECHNOLOGY

Paper – IV

Time : Three Hours] [Maximum Marks : 55

Note : Attempt *five* questions in all, selecting at least *one* question from each unit. Question No. 1 is compulsory.

Compulsory Question

1. (a) Find out the bandwidth of energy bands for simple cubic crystal from energy dispersion relation within Tight Binding approximation. 3

(b) Point out, how Hatree-Fock Equations are the generalization of Hartree equations. 3

(c) Why do nanomaterials show different properties as compared to their bulk counterparts ? Give an example to show the same. 3

(d) What is resonant tunneling diode ? What type of materials can be used prepare these diode ? 2

UNIT–I

2. (a) State and explain de Haas van Alphen effect (dHvA). By taking suitable example, show the oscillation of magnetic susceptibility as a function of magnetic field. 5½

(b) If an electron is moving in uniform magnetic field, show that the area enclosed by the electron path are quantized in the units of $2\pi eB/\hbar$. 5½

3. What is cyclotron resonance ? Establish basic condition for the same. Describe in details the cyclotron resonance of electron in semiconductors and metals. 11

UNIT–II

4. What is Brownian motion ? Find out Langevin equations of motion for the Brownian particle. Ignoring background random force, find out the solution of this equation. 11

5. What is therminoic emission ? Derive Richardson-Dushman equation in terms of emission coefficient. 11

UNIT–III

6. (a) Considering the resonant tunneling through two identical barriers in series for 1D case, find out transition probability through the double barrier. Futher, evaluate the simplified form of this probability for small phase differences away from resonance. 8

   (b) Give schematic representation of Nanodot, Nanowire and Nanosheet with suitable examples. 3

7. (a) Discuss the transport in quantum wave guide structures with suitable example. 5½

   (b) How the transport in nanostructures is different from bulk counterparts ? Establish quantized conductance in nanostructures. 5½

UNIT–IV

8. (a) Draw schematics of surface emitting laser. How the laser design in it is different from other quantum well lasers ? Explain. 5½

   (b) Differentiate interband and intraband transitions. How the laser action in quantum cascade laser can be justified with the help of intraband phototransition. 5½

9. (a) What are the important properties of photodetector ? Explain schematic involved and principle of multiple quantum well photodetector. 5½

   (b) Describe the salient features of blue quantum well laser ? Give its principle for laser action. How is it different from ordinary laser ? 5½

Roll No. .... Total Pages : 3

MDE/M-16 **4259**

ELECTRONICS–II

Paper–V

Time : Three Hours] [Maximum Marks : 55

Note : Attempt *five* questions in all. Q. No. 1 is compulsory. Select *one* question from each unit. All questions carry equal marks.

Compulsory Question

1. (a) Explain the term 'Standard load' in logic gates. 3

(b) Define Sequential system. How does it differ from Combinational system ? 3

(c) Explain what is meant by mask-programming on ROM. 2

(d) What does the following program segment do ?

MVIA 39H

ADI 97H

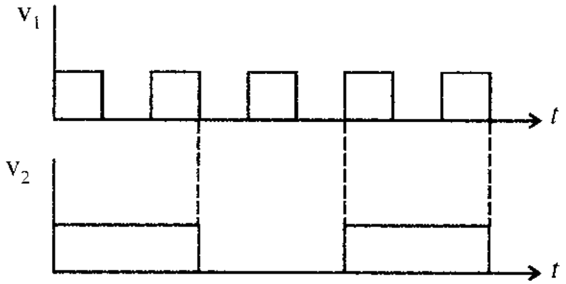
DAA 3

UNIT–I

2. (a) Explain Gray code. Elaborate the procedure to convert a binary code to gray code and vice-versa. 5

(b) Consider a two-input positive logic diode OR gate with following input waveforms. Sketch the output waveform if the ratio of the amplitudes of v_2 to v_1 is (i) 2 and (ii) ½.

Assume ideal diodes. 6



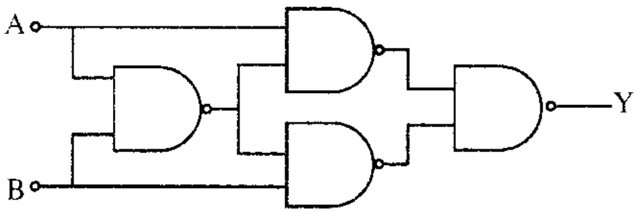
3. (a) Verify the following Boolean functions :

(i) $A\overline{B} + \overline{A}B + AB + \overline{AB} = 1$

(ii) $A\overline{B}D + AB\overline{D} + \overline{BD} = A\overline{B} + \overline{BD}$

(iii) $A\overline{B}(A + C) + AC(\overline{A} + \overline{C}) = A\overline{B}$. 5

(b) Briefly discuss XOR gate. Verify the following circuit is an XOR gate : 6



UNIT–II

4. (a) Simplify the expression using K-map : 6

$$f(A, B, C, D) = \sum 0, 3, 4, 5, 7 + \sum_{\phi} 9, 12, 13, 14, 15.$$

(b) Draw the circuit of a TTL gate, and explain its operation. 5

5. (a) Draw the circuit diagram of a ECL gate, and discuss its operation. Also list its advantages and disadvantages. 6

(b) Explain the operation of a 4-bit parallel binary full adder constructed from single bit full adders. 5

UNIT–III

6. Draw the block diagram of a 4-stage ripple counter and then sketch the output waveform from each flip-flop. Further explain how to modify it so that it divides by 10. 11

7. (a) Explain the operation of a four phase MOS shift register with suitable clock timing diagram. 6

(b) Show how to convert a JK-FF into a D-type FF. Verify its truth table. 5

UNIT–IV

8. Draw the pinout diagram of a 8085 microprocessor, and give a brief description of each pin. 11

9. (a) Discuss the architecture of 8085 microprocessor using the block diagram. 7

(b) Briefly explain the restart instructions in 8085 micro-processor. 4

Roll No. ....

Total Pages : 3

4260

OMDE/M-16

MATHEMATICS

(Advanced Abstract Algebra–II)

Paper : 406

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all, selecting at least *one* question from each section. All questions carry equal marks.

SECTION–I

1. (a) Prove that if $T \in A(V)$ has all characteristic roots in F, then T satisfies a polynomial of degree $n = \dim_F (V)$ over F.

(b) Prove that if $T \in A(V)$ is nilpotent, then T can be brought to a triangular form over F such that all the diagonal elements are zero. ($\dim_F(V) < \infty$)

2. (a) Prove that two nilpotent linear transformations on V ($\dim_F(V) < \infty$) are similar iff they have the same invariants.

(b) Prove that the matrix

$$\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in M_3(F)$$

is nilpotent.

Find its invariants.

3. (a) Let A, B $\in$ $M_n(F)$ be similar in $M_n(K) = K_n$, where $F \subseteq K$. Prove that A and B are already similar in $M_n(F)$.

(b) Let $T \in A_F(V)$ be such that $T^m = 1$. Prove that if Ch(F) = 0, and all the characteristic roots of T are in F, then T is diagonalisable.

SECTION–II

4. (a) Let R be a ring with unity. Prove that a left ideal I of R is a direct summand of R iff R is generated by an idempotent.

(b) Prove that if R is a ring with unity, then $Hom_R (R_R, R_R) \cong R$ as rings.

5. (a) Prove that if F is a free R-module having a basis of n elements, then $F \cong R^n$, $(1 \in R)$.

(b) Let $M = M_1 \oplus M_2$ be the direct sum of *nm*-isomorphic simple sub-modules M_1 and M_2 of M. Prove that $End_R(M)$ is a direct product of division rings.

SECTION–III

6. (a) Let N be a sub-module of a left R-module M such that both N and $\frac{M}{N}$ are Artinian R-modules. Prove that M is an Artinian R-module.

(b) Let R be a left Noetherian ring with 1. Prove that if $a, b, \in R$ be such that $ab = 1$, then $ba = 1$.

7. (a) Prove that a finite Boolean ring with unity is a finite direct product of binary fields.

(b) Prove that every non-zero sub-module of a Noetherian module contains a uniform module.

8. (a) Find the Abelian group generated by x, y, and z, where

$$5x + 9y + 5z = 0,$$
$$2x + 4y + 2z = 0,$$
$$x + y - 3z = 0.$$

(b) Prove with usual notations that if $M_m(D) \cong M_n(D')$, then $m = n$ and $D \cong D'$.

Roll No. .... Total Pages : 3

OMDE/M-16 **4264**

DIFFERENTIAL EQUATIONS–II

Paper : 410

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all, selecting at least *one* question from each section.

SECTION–I

1. (a) Describe Prüffer transformation in context of

$$\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0.$$ 8

(b) Prove that zeros of non-trivial solution of

$$(p(t)\, x'(t))'\, q(t)\, x(t) = 0,\ t \in \mathrm{I}$$

are isolated. 8

2. (a) State and prove Fundamental comparison theorem. 8

(b) Determine whether the equation

$$\frac{d^2y}{dx^2} + \left(\frac{2}{x^2} - \frac{1}{x^4}\right) y = 0;\ x > 1$$

is oscillatory or non-oscillatory ? 8

3. (a) Check the equation

$$x''(t) + \frac{2}{11\,t^2}\, x = 0$$

for being oscillatory or non-oscillatory. 8

(b) $u(t)$ and $v(t)$ are linearly independent solutions of $(p(t)x'(t))' + q(t)\, x(t) = 0$ on $[a,b]$ if and only if vectors $(u(t), pu')$ and (v, pv') are linearly independent solutions of HLS $x'(t) = A(t)\, x(t)$. Find $A(t)$ and prove above. 8

SECTION–II

4. (a) Define a Saddle point. Give an example. 4

(b) Given a system

$$\dot{x} = ax + by, \quad \dot{y} = cx + dy.$$

State the conditions when (0, 0) will be a node and prove that. 12

5. (a) Find the type and stability of the critical point of

$$\frac{dx}{dt} = x + 4y - x^2$$

$$\frac{dx}{dt} = 6x - y + 2xy.$$

(b) Define Limit cycle and Half path. Give example of each. 8

6. Given a dynamical system $m\frac{d^2x}{dt^2} = F(x)$ which is conservative. Establish the type and stability of critical point of its corresponding autonomous system in respect of its potential energy function

$$V(x) = -\int_0^x F(x)\, dx.$$ 16

SECTION–III

7. (a) Find eigen values and eigen function of SLBVP

$$\frac{d}{dx}\left(x\frac{dy}{dx}\right) + \frac{\lambda}{x} y = 0, \lambda \geq 0; \ y'(1) = y'(e^{2\pi}) = 0.$$ 8

(b) Describe construction of Green's function of a B.V.P.

$(p(t)\, x'(t))' + q(t)\, x(t) = 0;$

$x(A) + mx_1(A) = 0,$

$x(B) + nx'(B) = 0.$ 8

8. (a) Prove that Green's function of a second order linear boundary value problem is symmetric. 8

(b) Prove the orthogonality of characteristic functions of a SLBVP corresponding to its distinct eigen values. 8

Roll No. .... Total Pages : 3

MDE/M-16 **4297**

ADVANCED ABSTRACT ALGEBRA–II

Paper : MM-407

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each section. All questions carry equal marks.

Compulsory Question

1. Attempt all the following :

(a) Is Abelian group nilpotent or not ? Justify.

(b) Prove that

$$[xy, z] = [x, z]^y [y, z].$$

(c) When two linear transformations are said to be similar ?

(d) Define Index of nilpotence of nilpotent transformation.

(e) Define Cyclic module.

(f) Define Rank of a module.

(g) Show that $\mathbb{Z}$ as $\mathbb{Z}$ module is Noetherian.

(h) If V is n-dimensional vector space over a field F, then prove that V is Artinian.

SECTION–I

2. (a) Let G be a nilpotent group of class C, then prove that every factor group of G is again nilpotent group of class $\leq$ C.

(b) Is it true that if $K \Delta G$ such that both K and $\frac{G}{K}$ are nilpotent then G is nilpotent ? Justify your answer.

3. (a) Let G be a group and G′ be its commutator subgroup. Then prove that G′ is normal in G.

(b) If G is finite group and every Sylow p-group of G is normal, then prove that elements of coprime orders commute.

SECTION–II

4. (a) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, then prove that T satisfies a polynomial of degree n over F.

(b) If $u \in V_1$, where V_1 is subspace of V spanned by $u, T(u), \ldots, T^{n_1 - 1}(u)$ and $T^{n_1 - k}(u) = 0$ where $0 < k \le n_1$, then prove that $u = T^k(u_0)$ for some $u_0 \in V_1$.

5. (a) If F has characteristic $p > 0$, prove that

$$A = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix}$$

satisfies $A^p = I$.

(b) Prove that S and T in $A_F(V)$ are similar in $A_F(V)$ if and only if they have the same elementary divisors.

SECTION–III

6. (a) State and prove Fundamental theorem of R-homomorphisms.

(b) Let A and B be R-submodules of R-modules M and N, respectively. Prove that

$$\frac{M \times N}{A \times B} \simeq \frac{M}{A} \times \frac{N}{B}.$$

7. (a) Let $M = \sum_{\alpha \in \wedge} M_\alpha$ be a sum of simple R-submodules M_α. Let K be a submodule of M. Prove that there exists a subset $\wedge'$ of $\wedge$ such that $\sum_{\alpha \in \wedge'} M_\alpha$ is a direct sum, and

$$M = K \oplus \left(\oplus \sum_{\alpha \in \wedge'} M_\alpha \right).$$

(b) Let V be a non-zero finitely generated vector space over a field F. Then prove that V admits a finite basis.

SECTION–IV

8. (a) State and prove Hilbert basis theorem.

(b) If R is Noetherian, then prove that each ideal contains a finite product of prime ideals.

9. Prove that any right ideal of the ring $R = \begin{pmatrix} \mathbb{Z} & \mathbb{Q} \\ 0 & \mathbb{Q} \end{pmatrix}$ can be generated by atmost two elements, hence, R is right Noetherian.

(g) State (only) Jensen's inequality.

(h) Show that strict inequality in Minkowski's inequality, for $0 < p < 1$, may occur. (8×2=16)

Roll No. .... Total Pages : 4

MDE/M-16 **4298**

REAL ANALYSIS–II

Paper : MM-408

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all. Select *one* question each from Section-I to Section-IV while Q. No. 9 (Section-V) is compulsory.

SECTION–I

1. (a) Let $\{A_n\}$ be a countable collection of sets of real numbers. Show that

$$m^*\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} m^* A_n .$$

Hence, or otherwise, show that the outer measure of a countable set is zero. 8

(b) Let E be a given set. Show that E is measurable iff given $\varepsilon > 0$, there is an open set $0 \supset E$ with $m^*(0 \sim E) < \varepsilon$. 8

2. (a) Let C be a constant and f and g two measurable real value functions defined on the same domain. Show that the functions $f + c$, cf, $f + g$, $g - f$ and fg are also measurable. 8

(b) Give an example of a continuous function g and a meaurable function h such that hog is not measurable. 8

SECTION–II

3. (a) State and prove Lusin's theorem. 8

(b) Let f be defined and bounded on a measurable set E with mE finite. In order that

$$\inf_{f \le \psi} \int_E \psi(x)\, dx = \sup_{f \ge \phi} \int_E \phi(x)\, dx$$

for all simple functions ϕ and ψ, it is necessary and sufficient that f be measurable. 8

4. State and prove Lebesgue theorem regarding points of discontinuities of Riemann integrable functions.

SECTION–III

5. (a) State and prove Fatou's Lemma. Also show that we may have strict inequality in Fatou's Lemma. 8

(b) Show that the function

$f : [0, \infty) \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is not Lebesgue integrable over $[0, \infty)$. 8

6. (a) State and prove Vitali's Covering lemma. 10

(b) Show that a function of bounded variation is necessarily bounded, but not conversely. 6

SECTION–IV

7. (a) Let f be an integrable function on $[a, b]$, and suppose that

$$F(x) = F(a) + \int_a^x f(t)\, dt.$$

Show that $F'(x) = f(x)$ for almost all x in $[a, b]$. 8

(b) Show that a function F is an indefinite integral if and only if it is absolutely continuous. 8

8. (a) State and prove Holder's Inequality. 8

(b) Show that the L^p spaces are complete. 8

SECTION–V

(Compulsory Question)

9. (a) A set with outer measure zero is countable. Prove or disprove this statement.

(b) Give an example of a function which is not Lebesgue measurable.

(c) State (only) Egoroff's theorem.

(d) Give an example of a function which is Lebesgue integrable but not Riemann integrable.

(e) Give an example to show that the monotone convergence theorem need not hold good for a decreasing sequence of functions.

(f) State (only) Lebesgue differentiation theorem.

(d) What can be the meaning of x(5) in a FORTRAN-90 statement?

(e) Explain the working of functions ACHAR() and IACHAR() in FORTRAN-90 program.

(f) Give an example of using a derived data type into another.

(g) Explain the meaning of the following statement of FORTRAN-90.

INQUIRE(UNIT=20, OPENED=op_stat, NAME=file1)

(h) Describe the working of the function ASSOCIATED in FORTRAN-90.

Roll No. ....

Total Pages : 4

4299

MDE/M-16

COMPUTER PROGRAMMING (Theory)

Paper : MM-409

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all. Attempt *one* question from each of the Sections I to IV. Section V is compulsory. All questions carry equal marks.

SECTION–I

1. (a) Write the precedence order of various operators used in arithmetic expressions. Illustrate through example expressions. (4)

(b) Explain the domain allowed and working of the functions: LOG, LOG10, CMPLX, ABS, ASIN, COSH, TAN, SQRT. (8)

(c) Write a source program to convert a weight given in pounds into kilograms and grams. (4)

2. (a) Given three sides of a triangle, write program to check if it is an isosceles triangle or an equilateral triangle. (6)

(b) Explain the rules to be followed, while using the DO loop structures. (6)

(c) Write a source program to evaluate the sum $S = \Sigma(-1)^n x^{n/2}/n!$ for $n = 1$ to 10. (4)

SECTION–II

3. (a) Write source program, using SELECT CASE to sort the character (real, equal, complex) of the roots of a quadratic equation with given coefficients. (6)

(b) Use SELECT CASE in writing a source program to find the days in a given month of a given year. (10)

4. (a) Write a generalized syntax to define a function subprogram and discuss the rules to be followed in this definition. (10)

(b) Use internal function subprogram to verify

$$\sum_{m=1}^{n} m^3 = \left(\sum_{m=1}^{n} m\right)^2 . \qquad (6)$$

SECTION–III

5. (a) Use examples to illustrate the working of format rescan rule, in different situations. (8)

(b) Use example to illustrate the access to characters and parts in a given string. (4)

(c) Write a source program that combines the first name, middle name and surname to make the complete name for a person. (4)

6. (a) What is a palindrome? Write a source program, which uses a function subprogram to check a given string as a palindrome. (8)

(b) Use a derived data type for date, in writing a source program that checks whether an employee in a company is on the probation of one year or has become regular. (8)

SECTION–IV

7. (a) In the statement READ (5, 10, REC=x) x, y, z ; explain the significance of various keywords, constants and identifier used. (4)

(b) Write a source program to create a sequential file that contains the exam records (roll, name, marks of 5 subjects) of all the students in class. (6)

(c) Write a source program to read a matrix from one sequential file & to write its transpose to another sequential file. (6)

8. (a) Write a source program to create a linear linked list for a given set of characters. (8)

(b) Explain the use of NAMESLIST in input/output statements of FORTRAN-90 program. (4)

(c) Use COMPLEX data type to write a source program that illustrates the arithmetic operations between two complex numbers. (4)

SECTION–V

(Compulsory Question)

9. (a) What are requirements for a sequence of characters to be a valid identifier?

(b) Relate the functions INT() and MOD(), for their use in FORTRAN-90.

(c) Write a function subprogram to compute the magnitude of a given vector (x_1, x_2, x_3).

SECTION–V

(Compulsory Question)

9. (a) Define and explain Prüffer transformation.

(b) Give an example of non-oscillatory equation. Justify your claim.

(c) If two solutions of a 2nd order HLDE have a common zero, then will those two solutions be linearly independent ? State the result on which your deduction lies.

(d) Define Centre. Give *one* example.

(e) State Sturm Saparation theorem.

(f) Explain Limit cycle.

(g) Mention the situations when critical points of a non-linear system and its corresponding linear system are not alike.

(h) Define Singular boundary value problem. Construct an example with justification. (2×8=16)

Roll No. .... Total Pages : 4

MDE/M-16 **4301**

DIFFERENTIAL EQUATIONS–II

Paper : MM–411

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all. Select *one* question each from Section–I to IV. Question No. 9 (Section-V) is compulsory.

SECTION–I

1. (a) Every second order linear homogeneous differential equation can be transformed into an equivalent self-adjoint equation. Show the above by an example. 4

(b) State and prove Abel's formula. 8

(c) State the principle of Superposition. Check its validity for the differential eqn.

$$\frac{d^3y}{dx^3} + \sin x \frac{d^2y}{dx^2} + 7y\frac{dy}{dx} = 9x.$$ 4

2. (a) If $u(t)$ and $v(t)$ are solutions of $(p(t)\, y'(t))' + q(t)\, y(t) = 0$ on I, then prove that

$$\frac{1}{c}\int_{\tau}^{t} [u(s)\, v(t) - u(t)\, v(s)]\, h(s)\, ds, \; s \in \mathrm{I}$$

is a solution of $(py')' + qy = h(t)$.

Find $w(\tau)$ and $w'(\tau)$. 8

(b) Find a solution of the form $a_0 t^n + a_1 t^{n-1}$ of equivalent Riccoti equation of $(tp'(t))' + (1-t)\, p(t) = 0$. 8

SECTION–II

3. (a) Define and give examples of the following :

(i) Isolated critical point.

(ii) Spiral point. 4

(b) Consider a plane autonomous system and derive its characteristic equation. Find the type and stability of critical point(s) of that system when two roots of the ch. equation are real and equal. Prove your claim. 8

(c) What type of critical point (0, 0) of the system

$$\dot{x} = 2x + 4y,\ \dot{y} = -2x + 6y$$

will be ? Will the critical point be stable ? 4

4. (a) If $q(t) \in C\,(0, \infty)$ and $q(t) > 0$, $k > 0$, then prove that every real solution of $x''(t) + \left\{q(t) + k^2\right\} x(t) = 0$ has an infinite number of positive zeros. 8

(b) State and prove Hille-Wintner theorem. 8

SECTION–III

5. (a) Given a system

$$\dot{x}(t) = 3\cos x - 8\left(y + \frac{3}{8}\right)$$

$$\dot{y}(t) = \sin 2x - 5y\,.$$

Determine critical points, their type and stability. 8

(b) Determine the stability of the system

$$\dot{x}(t) = -3x + x^3 + 2xy^2,\ \dot{y}(t) = -2y + \frac{2}{3}y^3$$

by Liapunov method. 8

6. Write equation of rectilinear motion of a particle of mass m under the action of restoring force $F(x)$, x is displacement. Derive its equivalent autonomous system and let $(x, 0)$ be a critical point of that system. If $V(x) = -\int_0^x F(x)\,dx$, then prove that (i) if $V(x)$ has a relative minimum at $x = x_c$, then $(x_c, 0)$ is a centre and stable, (ii) if $V(x)$ has relative maximum at $x = x_c$, then $(x_c, 0)$ is saddle point and is unstable, (iii) if $V(x)$ has horizontal inflexion point at $x = x_c$, then $(x_c, 0)$ is a degenerative cusp and unstable. 16

SECTION–IV

7. (a) Solve the Boundary value problem

$$\frac{\partial u}{\partial t}(x, t) = k\frac{\partial^2 u}{\partial x^2}(x, t),\ 0 < x < 1,\ t > 0$$

$$\frac{\partial u}{\partial x}(0, t) = 0,\ \frac{\partial u}{\partial x}(1, t) = -hu(1, t)$$

$$u(x, 0) = f(x).$$ 8

(b) Define a Sturm-Liouville Boundary value problem and then prove that its eigen values are always real. 8

8. (a) Explain the construction of Green's function of a boundary value problem. 8

(b) Prove that upper bound of the error due to transaction of successive approximation at the nth stage is given by

$$\| x_n - x \| < \frac{p^n}{1 - p}\ \| x_1 - x_0 \|,$$

where symbols have usual meanings. 8

Roll No. .... Total Pages : 2

MDQ/M-16 **4409**

MATERIAL SCIENCE–II

Paper–I

Time : Three Hours] [Maximum Marks : 55

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each unit.

Compulsory Question

1. Attempt all the following :

(a) What is Silsbee law? 3

(b) What is coherence length for electrons in superconductors? 3

(c) Define Elastomers. 2

(d) What is Reinforced concrete? 2

UNIT–I

2. (a) What is Superconductivity? Describe the effect of magnetic field and frequency on superconductors. 7

(b) The critical temperature T_c for mercury with isotopic mass 199.5 is 4.185 K. Calculate its critical temperature when its isotopic mass changes to 203.4. 4

3. (a) From the consideration of entropy of superconducting and normal states show that the superconducting state is more orderly than normal state. 5

(b) Discuss the existence of energy gap parameters in superconductor. 6

UNIT–II

4. How are cooper pairs formed? Explain the BCS theory of superconductivity, and discuss energy gap based on this theory. 11

5. (a) What is the structure and chemical aspect of La-Ba-Cu-O superconductor? 5

(b) Show that a.c. Josephson's current oscillates with frequency $\omega = 2\ eV/h$. 6

UNIT–III

6. (a) The molecular weight of vinyl chloride is 62.5. Calculate the molecular weight of polyvinyl chloride with a degree of polymerization 20,000. 4

(b) What are Thermosets and Thermoplasts? 7

7. What are Amorphous and Crystalline polymers? How crystalline polymers respond to stress over temperature range ? 11

UNIT–IV

8. What are Ceramics? Discuss in detail their physical properties. 11

9. Discuss the various mechanical models for behaviour of polymers. 11

Roll No. .... Total Pages : 2

MDQ/M-16 **4410**

APPLIED NUCLEAR TECHNIQUE

Paper–II

Time : Three Hours] [Maximum Marks : 55

Note : Attempt *five* questions in all, selecting *one* question from each unit. Q. No. 1 is compulsory.

Compulsory Question

1. (a) Explain the principle of Tandem accelerator. 2

(b) Discuss the contribution of bremsstrahlung from the incident particles to the continuous background. 2

(c) Differentiate between X-ray emission and X-ray fluorescence. 3

(d) Calculate the short wave length limit for an X-ray tube operated at 50 kV. 2

(e) Define sensitivity of neutron activation analysis technique. 2

UNIT–I

2. (a) Differentiate between cyclotron and synchrotron. 3

(b) Discuss principle, construction and working of a cyclotron. Also discuss the limitations of energy that can be obtained by this machine and its possible improvement. 8

3. (a) What is a betatron? Derive the betatron condition for successful acceleration of electrons. 5

(b) Give constructional details of a proton synchrotron. Explain its working and theory. 6

UNIT–II

4. (a) Explain fundamentals of X-ray emission and Moseley's law. 5

(b) Draw a block diagram of P1XE set up. Explain the working of various parts. 6

5. (a) List the advantages of external beam PIXE over normal method. 5

(b) Discuss the applications of PIXE in environmental analysis. 6

UNIT–III

6. (a) Draw a schematic diagram of X-ray tube and explain the production of X-rays. 5

(b) Explain the working of wavelength dispersive devices. 6

7. Explain various aspects of elemental analysis of a sample using XRF spectrometer. 11

UNIT–IV

8. (a) Explain in detail the theory of activation method. 5

(b) Discuss classification of neutron activation methods. 6

9. (a) Describe in detail various standardization methods for neutron activation analysis technique. 6

(b) Discuss the application of NAA technique in soil science. 5

9. (a) Evaluate $\int_0^1 \frac{1}{1+x}\, dx$ correct to three decimal places by both the Trapezoidal and Simpson's 1/3rd rule with $h = 0.125$. 6

(b) Describe Legendre-Gauss quadrature to evaluate definite integral numerically. Give its geometrical interpretation. 5

Roll No. ....

Total Pages : 4

MDQ/M-16 **4411**

COMPUTATIONAL PHYSICS

Paper–III

Time : Three Hours] [Maximum Marks : 55

Note : Attempt *five* questions in all. Q. No. 1 is compulsory. Select *one* question from each unit.

Compulsory Question

1. (a) Write the FORTRAN expressions for the following :

(i) $\frac{a}{c+d} \times \frac{e}{f}$

(ii) $\sin \left|\frac{x-y}{x+y}\right|$

(iii) $\sqrt{\log\ (|2x|)}$. 3

(b) Differentiate between Round-off error and Truncation error. 2

(c) What is an ill-conditioned matrix? 3

(d) Show that Simpson's 1/3rd rule of integration is more accurate than the trapezoidal rule. 3

UNIT–I

2. (a) Give brief description of various Input and Output devices. 5

(b) Discuss the following with example :

(i) FORMATTED INPUT and OUTPUT statements.

(ii) NESTED IF statement.

(iii) COMMON statement. 6

3. (a) How arrays and subscripted variables are treated in FORTRAN? Explain with examples. 5

(b) Using the concept of SUBROUTINE subprogram, write a FORTRAN program to find the roots of a quadratic equation. 6

UNIT–II

4. (a) Discuss the propagation of error in multiplication and division. 4

(b) Describe Bisection method to find the roots of equation $f(x) = 0$ and hence find a real root of the equation $f(x) = x^3 + x^2 - 1 = 0$ correct to three decimal digits. 7

5. (a) Write a FORTRAN program to find the roots of equation $f(x) = 0$ by using Newton-Raphson method. 5

(b) Find the value of $f(0.22)$ using the following data by employing a suitable interpolation formula :

t :	0.0	0.2	0.4	0.6	0.8	1.0
$f(t)$:	1.1	1.016	0.936	0.84	0.724	0.59

6

UNIT–III

6. (a) Write a FORTRAN program to multiply two matrices of order $n \times m$ and $m \times n$. 5

(b) Find the inverse of the following matrix :

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 2 & 5 \\ 1 & -1 & 0 \end{bmatrix}.$$ 6

7. (a) Describe Gauss-Jordan elimination method to solve a set of simultaneous linear algebraic equations. 5

(b) Using Power method, find the largest eigen value for the matrix A given by

$$A = \begin{bmatrix} 9 & 7 & 1 \\ 1 & 4 & 7 \\ 8 & 1 & 9 \end{bmatrix}.$$ 6

UNIT–IV

8. (a) Describe Taylor's series method of numerical differentiation for computing first and second order derivatives. Explain with suitable examples. 5

(b) Using the value of $h = 0.05$, find the first and second order derivatives of the function $f(x) = 1 + x^2$ at $x = 0.6$ using Newton's Forward difference formula. 6

Roll No. .... Total Pages : 2

4412

MDQ/M-16
FIBRE OPTICS
Paper–IV

Time : Three Hours] [Maximum Marks : 55

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each unit. All questions carry equal marks.

Compulsory Question

1. (a) What is the criterion for the selection of cables in Optical fibre communication ?

(b) How refractive index is measured for optical fibres ?

(c) Explain Step index fibres.

(d) What are Trans receivers ?

UNIT–I

2. Explain the propagation of light in optical fibres in detail. Discuss the basic structure and optical path of an optical fibre.

3. Explain modes of propagation, meridional and skew rays including number of modes and cut-off parmaters of fibre.

UNIT–II

4. How fibres are fabricated using outside vapour phase oxidation and vapour phase axial technique ? Explain.

5. Give classification of optical fibres as stepped index fibre, Stepped index monomode fibre and Graded index multimode fibres. What are the disadvantages of monomode fibres?

UNIT–III

6. Explain how scattering losses, dispersion losses and cut-off wavelength of optical fibres is measured.

7. What do you understand by Near field scanning technique ? Give its indirect method. Also explain Transverse offset technique and Variable aperture technique.

UNIT–IV

8. Explain LED analog transmitter, and compare between analog and optical transmitter alongwith their applications.

9. Explain the block diagram of fibre optical receiver. What is high performance receiver ? Explain the role of repeaters in optical fibre communication.

Roll No. .... Total Pages : 2

MDQ/M-16 **4413**

COMMUNICATION SYSTEM

Paper–V

Time : Three Hours] [Maximum Marks : 55

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each unit. All questions carry equal marks.

Compulsory Question

1. (a) What are the characteristics of Data transmission system in digital communication ?

(b) Differentiate between different Modulation schemes.

(c) What are the noise considerations in Optical fibre communication ?

(d) What are the different standards in case of TV ?

UNIT–I

2. What do you understand by Pulse communication ? Discuss in detail PAM and PPM alongwith their applications.

3. What is the model of Communication system ? Discuss its analysis and design.

UNIT–II

4. Describe in detail the spectral analysis of modulation and demodulation operations.

5. What is Randon signal theory ? Discuss information and channel capacity.

UNIT–III

6. What do you understand by Error ? Explain error control coding. Discuss one method to control errors.

7. What is Satellite communication ? In case of Satellite communication explain orbits, station keeping, transmission path losses and satellite altitude.

UNIT–IV

8. What do you understand by Point-to-point communication ? Describe Telephone networks and explain the working of automatic exchange switching systems.

9. What is RADAR ? Explain Primary radar and Secondary surveillance radar in detail. What are its applications ?

Roll No. ....   Total Pages : 3

4545

MDQ/M-16

MATHEMATICS

(General Measure and Integration Theory)

Paper : MM-507

Time : Three Hours]   [Maximum Marks : 80

Note : Attempt *five* questions in all. Select *one* question each from Section-I to Section-IV. Q. No. 9 (Section-V) is compulsory.

SECTION–I

1. If v is an outer measure on a hereditary σ-ring $\mathcal{H}$, and $\mathcal{M}$ is the class of all v-measurable sets, then show that

(a) $\mathcal{M}$ is a σ-ring,

(b) If E_n is a sequence of mutually disjoint sets in $\mathcal{M}$, whose union is E, then $v(A \cap E) = \sum_{n=1}^{\infty} v(A \cap E_n)$ for every A in $\mathcal{H}$,

(c) The restriction of v to $\mathcal{M}$ is a measure. 16

2. (a) If f is measurable, and $\phi : R \to R$ is a Borel measurable function such that $\phi(0) = 0$, then show that the composite function $\phi o f$ is also measurable. 8

(b) If f is a measurable function, and A is a locally measurable set, then show that the function $\chi_A f$ is also measurable. 8

SECTION–II

3. (a) State and prove Arzela-Young theorem. 6

(b) State and prove Riesz-Weyl theorem. 10

4\. (a) If $\{f_n\}$ is a sequence of ISF such that $f_n \downarrow 0$, then show that $I(f_n) \downarrow 0$. 8

(b) If f is integrable, g is measurable, and $f = g$ a.e., then show that g is also integrable, and

$$\int f \, d\mu = \int g \, d\mu .$$ 8

SECTION–III

5\. State and prove Fubini's theorem. 16

6\. (a) If (X, ζ, μ) is a measure space, and v is a finite measure on ζ, show that the following conditions on v are equivalent :

(i) v is AC with respect to μ.

(ii) $\mu(E) = 0$ implies $v(E) = 0$. 8

(b) State and prove Jordan-Hahn decomposition theorem. 8

SECTION–IV

7\. (a) If $C \subset U$, where C is compact and U is open, show that there exists an f in £ such that $0 \leq f \leq 1, f = 1$ on C, and $f = 0$ on $X - U$. 8

(b) Show that every Baire measure is regular. 8

8\. (a) If μ_1 and μ_2 are regular Borel measures on X, and T is a homeomorphism of X such that $\mu_1(T^{-1}(D)) = \mu_2(D)$ for every compact $G_s D$, show that

$$\int f^T \, d\mu_1 = \int f \, d\mu_2 \text{ for all } f \text{ in £.}$$ 8

(b) If ϕ is a positive linear form on £, show that there exists a unique regular Borel measure μ such that

$$\phi(f) = \int f \, d\mu \text{ for all } f \text{ in £.}$$ 8

SECTION–V
(Compulsory Question)

9\. Attempt all the following :

(a) Show that an additive positive set function v defined on a ring is necessarily monotone.

(b) If f is measurable, show that the function $\frac{|f|}{(1+|f|)}$ is measurable.

(c) State (only) Egoroff's theorem.

(d) Convergence a.e. does not always imply convergence in measure. Prove or disprove this statement.

(e) State (only) Radon Nikodym theorem.

(f) State (only) Baire Sandwich theorem.

(g) Define a Baire measure.

(h) What do you mean by a measurable rectangle ?

(8×2=16)

Roll No. .... Total Pages : 3

MDQ/M-16 **4546**

PARTIAL DIFFERENTIAL EQUATIONS

Paper : MM-508

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all. Question No. 1 is compulsory. Select *one* question from each section. All questions carry equal marks.

Compulsory Question

1. (a) What do you mean by Well posed problem ?

(b) Write the solution of non-homogeneous transport equation.

(c) Obtain $D\Phi(y)$, where $\Phi(y)$ is the fundamental solution of Laplace equation in $U \subset R^n$.

(d) Verify that $\Phi(y - \tilde{x})$ is the character function in

$U = \{x \in R^n_+\}$ where $\tilde{x} = \{x_1, x_2, ..., x_n\}$.

(e) Give the physical interpretation of D′ Alemberts formula.

(f) Write the characteristic equations of

$$x_1 u_{x_2} - x_2 u_{x_1} - u = 0 \text{ in } U \subset R^2.$$

(g) Explain convex duality of Hamiltonian and Lagrangian.

(h) Write the properties of Fourier transform.

SECTION–I

2. (a) Solve

$$u_t + b \cdot Du = f \quad \text{in } R^n \times (0, \infty)$$
$$u = g \quad \text{on } R^n \times \{t = 0\}.$$

(b) State and prove the strong maximum principle for harmonic function.

3. If u is harmonic in $U \subset R^n$, then prove that u is analytic in U.

SECTION–II

4. (a) Derive the Green's function for the region $U \subset R^n$.

(b) Derive the fundamental solution of heat equation.

5. (a) Use energy method to prove that

$$u_t - \Delta u = f \quad \text{in } U_T$$
$$u = g \quad \text{on } \Gamma_T$$

has a unique solution.

(b) Solve

$$\Delta u = 0 \quad \text{in } B^o(0, r)$$
$$u = g \quad \text{on } \partial B(0, r);$$

using Green's representation formula.

SECTION–III

6. (a) Solve

$$u_{tt} - u_{xx} = 0 \quad \text{in } R_+ \times (0, \infty)$$
$$u = g,\ u_t = h \quad \text{on } R_+ \times \{t = 0\}$$
$$u = 0 \quad \text{on } \{x = 0\} \times (0, \infty)$$

with $u(0) = h(0) = 0$.

(b) Derive Euler's-Poisson-Darboux equation from

$$u_{tt} - \Delta u = 0 \quad \text{in } R^n \times (0, \infty)$$
$$u = g,\ u_t = h \quad \text{on } R^n \times \{t = 0\}$$

$n \geq 2$.

7. (a) Solve

$$u_{x_1} u_{x_2} = 0 \quad \text{in } U = \{x_1 > 0\}$$
$$u = x_2^2 \quad \text{on } \pi = \{x_1 = 0\}$$

(b) Prove that

$$u(x, t, a, b) = a \cdot x - t\, H(a) + b,$$

$a \in R^n$, $b \in R$, is a complete integral of the Hamilton-Jacobi equation

$$u_t + H(Du) = 0.$$

SECTION–IV

8. (a) Use separation of variables to find a non-trivial solution of

$$u_{x_1}^2 u_{x_1x_1} + 2u_{x_1}u_{x_2}u_{x_1x_2} + u_{x_2}^2 u_{x_2x_2} = 0 \text{ in } R^2.$$

(b) If $u \in L^1(R^n) \cap L^2(R^n)$, then prove that

$$\|\hat{u}\|_{L^2(R^n)} = \|u\|_{L^2(R^n)} = \|u\|_{L^2(R^n)}.$$

9. (a) Solve

$$u_t - \Delta(u^r) = 0 \text{ in } R^n \times (0, \infty)$$

where $u \geq 0$, and $r > 1$ is a constant for scaling invariant solution.

(b) Obtain travelling wave solution of

$$u_t + 6uu_x + u_{xxx} = 0 \text{ in } R \times (0, \infty).$$

Roll No. ....   Total Pages : 3

MDQ/M-16   **4547**

MECHANICS OF SOLIDS – II

Paper : MM-509(i)

Time : Three Hours]   [Maximum Marks : 80

Note : Attempt *five* questions in all. Select *one* question each from Section-I to Section-IV. Question No. 9 (Section-V) is compulsory. All questions carry equal marks.

SECTION–I

1. (a) Explain Plane deformation. Obtain the relevant field equations for plane strain.

(b) Find the boundary conditions for the boundary value problem when the normal and tangential components of stress vector are given on the boundary.

2. Show that for a plane strain problem the stresses and displacements can be expressed in terms of two analytic function.

SECTION–II

3. (a) Show that in a homogeneous, isotropic, elastic medium *two* types of waves propagate.

(b) Derive the equation giving the velocity of propagation of Rayleigh waves.

4. Solve the problem of bending of a beam by terminal couples.

SECTION–III

5. Solve the torsion problem of a circular shaft twisted by a couple M.

6. (a) Obtain the conjugate hormonic function ϕ for an elliptic section.
   (b) A circular shaft of length C, radius a and shear modulus μ is twisted by a couple M. Show that the greatest angle of twist θ and the maximum shear stress T are given by

$$\theta_{max} = \frac{2MC}{\pi \mu a^4}, \quad T_{max} = \frac{2M}{\pi a^3}.$$

SECTION–IV

7. State and prove the theorem of Minimum potential energy.

8. (a) Discuss the deflection of the central line of a beam.
   (b) Use Galerkin method to find an approximate solution of the problem

$$\nabla^2 \psi = -2 \text{ in R}$$

$\psi = 0$ on the boundary of R, where R is the rectangle

$$|x| \le a, \ |y| \le b.$$

SECTION–V

Compulsory Question

9. Answer all the following :
   (a) Define Plane stress.
   (b) State Second boundary Value problem in plane elasticity.

   (c) Define Plane waves.
   (d) Define Love waves.
   (e) State Torsion problem of cylindrical bars.
   (f) Define Prandtl stress function.
   (g) State Reciprocal theorem of Betti and Rayleigh.
   (h) State Kentrovich method.

Roll No. .... Total Pages : 3

MDQ/M-16 **4551**

FLUID MECHANICS–II

Paper : MM-510

Opt. (i)

Time : Three Hours] [Maximum Marks : 80

Note : Attempt five questions in all. Question no. 9 is compulsory. Select *one* question from each Section. All questions carry equal marks.

SECTION–I

1. (a) State and prove Milne-Thomson circle theorem and apply it to find the image of a doublet relative to a circle.

(b) Derive equation of continuity in the cylindrical coordinate system and hence deduce the condition for the incompressible fluid flow.

2. (a) A circular cylinder of density σ moves with velocity U in an infinite mass of liquid of density ρ at rest at infinity, then find the ratio in which effect of external forces are reduced due to presence of the liquid.

(b) Derive the complex potential function for source and doublet. Also find image of source with respect to a circle.

SECTION–II

3. (a) State and prove Kutta-Joukowski theorem.

(b) Derive kinetic energy of liquid contained rotating elliptic cylinder.

4. (a) Derive stream function and velocity component for the combination of a uniform stream and a point doublet at the origin.

(b) Show that there cannot be two different forms of acyclic irrotational motion of a given liquid whose boundaries have prescribed velocities.

SECTION–III

5. (a) Find stream function and potential function for the liquid streaming past a fixed sphere.

(b) State and prove Kelvin's proof of permanence theorem.

6. (a) In an infinite liquid n rectilinear vortices of the same strength K are symmetrically arranged along generators of a circular cylinder of radius 'a' and prove that the vortices will move round the cylinder uniformly in time

$$\frac{4\pi a^2}{K(n-1)}.$$

(b) State and prove Karman vortex street theorem.

SECTION–IV

7. (a) Obtain the Prandtl's boundary layer approximation to the Navier-Stokes equation for steady two dimensional flow.

(b) Obtain Blasius solution to the boundary layer equation.

8. (a) State Buckingham π-theorem and discuss the main steps of Buckingham π-theorem in deriving dimensionless π-terms.

(b) Discuss separation of boundary layer for the steady flow over a flat plate.

Compulsory Question

9. Answer the following :

(a) Define acyclic irrotational motion of a given liquid.

(b) Define uniform flow in tho x-direction, y-direction and at an angle α to the x-direction.

(c) Define boundary layer and its thickness.

(d) Define source sink and doublet.

(e) Define potential flow.

(f) What is physical meaning of div $\bar{q} = 0$?

(g) Define circulation and write the mathematical relation that connect it with the vortices.

(h) Write mathematical expression for Reynolds number and explain its physical importance.

(e) Find the resolvent kernel for the equation

$$g(s) = f(s) + \int_0^s (s-t)\, g(t)\, dt. \qquad 2$$

(f) Define finite Hilbert transform pair. 2

(g) Write the value of $R(s,t)$. 2

(h) What is the difference between Dirichlet and Neumann problems. 2

B

Roll No. .... Total Pages : 4

MDQ/M-16 **4552**

BOUNDARY VALUE PROBLEMS

Paper : MM-510

Opt. (ii)

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all, selecting *one* question from each section. Question No. 9 is compulsory .

SECTION–I

1. Solve the initial :

(a) Value problem

$y'' + A(s)\, y' + B(s)y = F(s)\ y(a) = q_0,\ y'(a) = q_1.$ 8

(b) Find the deflection $y(s)$ that is parallel to the y–axis and satisfies the system of equations :

$$\frac{d^4y}{ds^4} - k^4 y = 0,\ k^4 = \frac{w^2 d}{EI}$$

$$y(0) = y'(0) = 0$$

$$y''(l) = y'''(l) = 0$$ 8

2. Explain and modified Green's function and obtain it for the boundary value problem

$$\frac{d}{ds}\left\{(1-s^2)\frac{dy}{ds}\right\} + \lambda y = 0$$

$y(-1), y(1)$ } finite. 16

SECTION–II

3. (a) Present an integral equation formulation of the exterior and interior Dirichlet boundary value problem in a composite medium. 8

(b) Solve the Helmholtz equation :

$(\nabla^2 + \lambda)\vec{u} = 0$ with $\vec{u}/s = \tau$ and

$$\left.\frac{\partial \vec{u}}{\partial n}\right|_s = \sigma .$$ 8

4. Obtain the Acoustic diffraction of a plane wave by a perfectly soft disk.

$u(\rho, \phi, z) = u_i(\rho, \phi, z) + u_s(\rho, \phi, z)$

$(\nabla^2 + k^2)\, u_s = \theta$

$u_i(\rho, \phi, 0) + u_s(\rho, \phi, 0) = 0$

$0 \le \rho \le a.\ u_s$ and

$\frac{\partial u_s}{\partial z}$ are continuous across $z = 0$, $a < \rho < \infty$. 16

SECTION–III

5. (a) Solve the integral equation

$$f(s) = \int_0^s \frac{g(t)}{(s^2 - t^2)\alpha}\, dt$$

using Laplace transform. 8

(b) Constitute a second form of the finite Hilbert transform pair using first form pair. 8

6. Explain two-part boundary value problem and solve the integral equation

$$\int_0^a t\phi(t) \int_0^\infty J_1(P\rho)\, J_1(Pt)\, d\, P dt = \Omega\rho ,\ 0 < \rho < a.$$ 16

SECTION–IV

7. (a) Find a solution of the Fredholm integral equation of the first kind

$f(P) = \int K(P,Q)\, g(Q)\, ds$, $P \in S$ using perturbation techniques. 8

(b) Explain the application of perturbation methods to electrostatics. 8

8. Find the torque experienced by a sphere which is rotating uniformly in Oseen flow and is bounded by a pair of parallel walls $z = \pm c$. Evaluate also the velocity field. 16

SECTION–V

Compulsory Question

9. (a) When we solve boundary value problem it leads to .... type integral equations. 2

(b) Write two properties of modified Green's functions. 2

(c) Show that the Green's function satisfying the equation

$-\nabla^2 G = \delta(\vec{x} - \vec{\xi})$, $G(\vec{x}\,;\vec{\xi})/s = 0$ is symmetric. 2

(d) Prove that :

$$G(P) = \frac{F(P)}{1 - K(P)} .$$ 2

Roll No. .... Total Pages : 3

MDQ/M-16 **4555**

MATHEMATICAL ASPECTS OF SEISMOLOGY

Paper : MM-511

Opt. (i)

Time : Three Hours] [Maximum Marks : 80

Note : Attempt *five* questions in all. Select *one* question each from Section-I to Section-IV. Q. No. 9 (Section-V) is compulsory.

SECTION–I

1. (a) Obtain progressive type solution of one-dimensional wave equation using D'Alembert's method.

(b) What is the wavelength and velocity of the system of plane waves $\phi = a \sin (Ax + By + Cz - Dt)$? (10+6)

2. (a) Find stationary type solution of wave equation in spherical coordinates.

(b) Obtain the relation between Phase velocity and Group velocity. (10+6)

SECTION–II

3. Define P and S waves of Seismology. Discuss the reflection of P-waves at a plane free surface of a semi-infinite solid medium. (16)

4. What do you mean by Love waves and Stanely waves ? Derive the frequency equation for Love waves in a layer of uniform thickness overlying a uniform half space. Explain whether Love waves are dispersive. (16)

SECTION–III

5. Find the solution of a two-dimensional Lamb problem when the normal force is applied on the surface of a semi-infinite elastic solid. (16)

6. Discuss Haskell-matrix method to solve the problem of Love waves in a multilayered medium with plane boundaries. (16)

SECTION–IV

7. (a) Describe expansion of spherical waves into plane waves.
   (b) Derive Poisson's formula. (8+8)

8. (a) Write a short on Aftershocks and Foreshocks of Earthquakes.
   (b) Describe Seismic moment and Earthquake magnitude. (8+8)

SECTION–V

(Compulsory Question)

9. Attempt all the following :
   (a) Define Plane waves.
   (b) Show that three equivalent harmonic waves with 120° phase between each pair have zero sum.

   (c) Explain Reflection at critical angle.
   (d) Define Snell's law of reflection and refraction.
   (e) Define Isotropic elastic solid.
   (f) Define Line source and Point source.
   (g) What do you mean by Richter scale ?
   (h) What is Crust and Mantle of the earth ? (8×2=16)