Minimum Noise during Wave Mixing Spectroscopy in High Order Susceptibility materials

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ABSTRACT

It is well known that when focussed laser beam interacts with a material of high order susceptibility, higher order wave mixing spectroscopy becomes significant. This paper presents the interaction of three pump photons at frequency ω_1 with three emitted photons at different frequencies ω_2, ω_3 and ω_4, respectively in a non- linear material during six wave mixing process. During interaction, it has been found that the value of quantum noise in field amplitude of pump mode reduces than its minimum value depending on the selective phase values of field amplitude along with sub- Poissonian photons statistics. The variation of field amplitude variance with number of photons has also been shown graphically.

Key- words: Quantum squeezing, Photons statistics, Wave- mixing spectroscopy, Non-linear materials etc.

1. Introduction:

In Quantum mechanics, the term squeezing refers to reduced quantum uncertainty. The state of an electromagnetic field is said to be squeezed when the noise in one variable is reduced below the standard limit at the expense of the increased noise in the conjugate variable such that the Heisenberg uncertainty relation is not violated. The noise in quantum mechanics sets a standard limit on the transmission of a signal and precision measurements in interferometers. The uncertainty is minimum and equally distributed over the two conjugate variables for a coherent state. A lot of work has been presented in literature on the generation of squeezed states experimentally and theoretically [1-15]. Due to less noise, such states have found potential applications in gravitational wave detection [16,17], teleportation [18,19], cryptography [20], precision measurement interferometers[21,22], quantum sensors[23] etc.

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The paper investigates the generation of squeezed states during six wave interaction spectroscopy under short time approximation, theoretically. Squeezing occurs for selective phase values of the field amplitude of the pump mode at frequency ω_1. The statistical behaviour of photons in pump mode has also been studied.

2. Condition of squeezing

Consider a single mode of electromagnetic field of frequency ω with creation and annihilation operators $a^\dagger$ and a respectively. Amplitude squeezing is defined in terms of the slowly varying operators A and $A^\dagger$ defined as

$$X_1=1/2\,(A+A^\dagger) \text{ and } \qquad X_2=1/2i(A-A^\dagger) \tag{1}$$

$$A=a \exp(\,i\omega t) \qquad \text{and} \qquad A^\dagger=a^\dagger \exp(-i\omega t) \tag{2}$$

The operators obey the commutation relation

$$[\,X_1, X_2] = i/2 \tag{3}$$

This leads to the uncertainty relation

$$\Delta X_1 \Delta X_2 \geq 1/4 \tag{4}$$

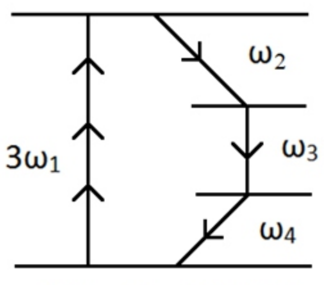
A quantum state is said to be squeezed in X_i variable if

$$[\Delta X_i]^2 \leq 1/4 \text{ for } i=1 \text{ or } 2. \tag{5}$$

Here, equality sign refers to coherent state.

3. Squeezing in pump mode at frequency ω_1 during six wave mixing

The six wave interaction model is shown in figure. Three pump photons at frequency ω_1 are absorbed by a nonlinear high susceptibility material which cause subsequent emission of three photons at frequencies ω_2, ω_3, ω_4, respectively.



$$3\omega_1 = \omega_2 + \omega_3 + \omega_4$$

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The Hamiltonian for this process is given as follows ($\hbar$=1)

$$H=a^{\dagger}a\omega_1+ b^{\dagger}b\ \omega_2+ c^{\dagger}c\ \omega_3+d^{\dagger}d\ \omega_4+g\ (a^3b^{\dagger}c^{\dagger}\ d^{\dagger}+c.c.) \quad (6)$$

The energy transfer equation is

$$3\omega_1=\omega_2+ \omega_3+ \omega_4$$

The Heisenberg equation of motion for the mode A is

$$dA/dt=\partial A/\partial t+i[H,A] \quad (7)$$

Using eqn. (6) in eqn.(7), we obtain

$$dA/dt= -3igA^{\dagger 2}BCD \quad (8)$$

Similarly, we obtain the relations for dB/dt , dC/dt and dD/dt as

$$dB/dt= -igA^3C^{\dagger}\ D^{\dagger} \quad (9)$$

$$dC/dt= -igA^3B^{\dagger}D^{\dagger} \quad \text{and} \quad (10)$$

$$dD/dt= -igA^3B^{\dagger}C^{\dagger} \quad (11)$$

With the assumption of small interaction time during the process, expanding A(t) in Taylor's series and retaining the terms upto g^2t^2 (g^2t^2<<1), we obtain

$$A(t) = A - 3igtA^{\dagger 2}BCD + 3/2g^2t^2\,(6A^{\dagger}A^2B^{\dagger}BC^{\dagger}C\ D^{\dagger}D+ 6AB^{\dagger}BC^{\dagger}C\ D^{\dagger}D$$
$$-A^{\dagger 2}A^3\ C^{\dagger}C\ D^{\dagger}D-A^{\dagger 2}A^3\ D^{\dagger}D -A^{\dagger 2}A^3\ B^{\dagger}B\ D^{\dagger}D-A^{\dagger 2}A^3\ B^{\dagger}BC^{\dagger}C-A^{\dagger 2}A^3\ B^{\dagger}B$$
$$- A^{\dagger 2}A^3\ C^{\dagger}C - A^{\dagger 2}A^3) \quad (12)$$

The real quadrature component of the pump field is given as

$$X_{1A}(t) = ½[A\ (t) + A^{\dagger}(t)] \quad (13)$$

Initially, we consider the quantum state of the system as a product of coherent state for the mode A and vacuum states for the modes B, C and D

$$\text{i.e. } \psi|=\alpha|0|0|0| \quad (14)$$

The expectation values of real quadrarure component are derived as

$$\psi|X^2_{1A}(t)|\psi= ¼[\ \alpha^2+ \alpha^{*2}+ 2\,|\alpha|^2 +1+3/2g^2t^2(-2\alpha^2\,|\alpha|^4$$
$$- 2\alpha^{*2}\,|\alpha|^4 -2\alpha^2\,|\alpha|^2 - 2\alpha^{*2}\,|\alpha|^2- 4\,|\alpha|^6] \quad (15)$$

and

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$$\psi\ |X_{1A}(t)|\psi^2 = ¼[\ \alpha^2 + \alpha^{*2} + 2|\alpha|^2 + 3/2g^2t^2[-2\alpha^2|\alpha|^4 - 2\alpha^{*2}|\alpha|^4 - 4|\alpha|^6] \qquad (16)$$

Therefore

$$[\Delta X_{1A}(t)]^2 = X^2_{1A}(t) - X_{1A}(t)^2$$
$$=1/4[1+3/2g^2t^2(-2\alpha^2|\alpha|^2 - 2\alpha^{*2}|\alpha|^2) \qquad (17)$$
$$\Delta X_{1A}(t)]^2 - 1/4 = -3/4g^2t^2\ |\alpha|^2\ (\alpha^2+\alpha^{*2}) \qquad (18)$$
$$[\Delta X_{1A}(t)]^2 - 1/4 = \Delta S_A = -3/2g^2t^2|\alpha|^4 \cos 2\theta \qquad (19)$$

Equation (19) clearly shows that squeezing occurs in real quadrature component of field amplitude for selective phase values θ. Squeezing will be maximum for θ=0.

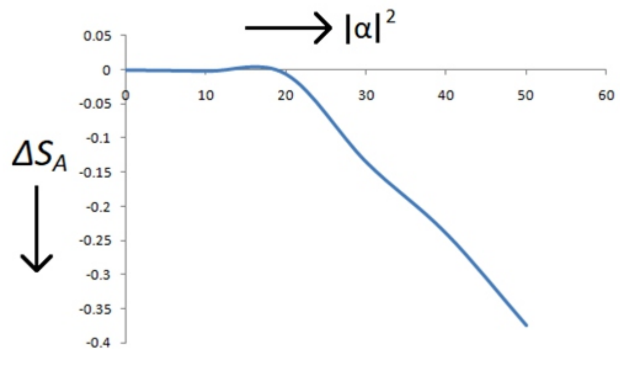
4. Photons- statistics: The fluctuation of photons in the pump mode A during six wave mixing process can be seen from the difference $\Delta n(t)^2 - n(t)$, which measures the departure from the Poissonian statistics. The negative value shows the sub-Poissonian behavior.

From eqns. (12) - (14), we have

$$\Delta\ n(t)^2 - n(t)$$
$$= n^2(t) - n(t)^2 - n(t)$$
$$= -6g^2t^2|\alpha|^6 = < 0 \qquad (20)$$

i.e., $\Delta\ n^2 < n$

The above expression (20) indicates that the squeezed light also observes the sub-Poissonian behavior photons. The variation of degree of squeezing with photon number is shown as



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5. Conclusion

The present paper shows one of the possible ways of generating quantum squeezed states in the pump mode depending upon the selective phase values of the field amplitude during six wave interaction spectroscopy in a material of high susceptibility. The squeezed light also exhibits the photons sub-Poissonian statistical behaviour. It has also been found that degree of squeezing increases with the increase in number of photons of pump mode.

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